

# Constraints on cosmic hemispherical power anomalies from quasars

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**Abstract.** Recent analyses of the cosmic microwave background (CMB) maps from the WMAP satellite have uncovered evidence for a hemispherical power anomaly, i.e. a dipole modulation of the CMB power spectrum at large angular scales with an amplitude of  $\pm 14$  percent. Erickcek *et al* have put forward an inflationary model to explain this anomaly. Their scenario is a variation on the curvaton scenario in which the curvaton possesses a large-scale spatial gradient that modulates the amplitude of CMB fluctuations. We show that this scenario would also lead to a spatial gradient in the amplitude of perturbations  $\sigma_8$ , and hence to a dipole asymmetry in any highly biased tracer of the underlying density field. Using the high-redshift quasars from the Sloan Digital Sky Survey, we find an upper limit on such a gradient of  $|\nabla\sigma_8|/\sigma_8 < 0.027r_{\text{ls}}^{-1}$  (99% posterior probability), where  $r_{\text{ls}}$  is the comoving distance to the last-scattering surface. This rules out the simplest version of the curvaton spatial gradient scenario.

## 1. Introduction

The simplest inflationary models for the origin of structure of the Universe has made a number of successful predictions, including the flatness of the Universe ( $\Omega_K = 0$ ), and near-scale invariance ( $n_s \approx 1$ ), adiabaticity, and Gaussianity of the primordial fluctuations. Another prediction of these models is that the perturbations should be statistically homogeneous and isotropic. It was therefore a surprise when, after the cosmic microwave background (CMB) maps from the Wilkinson Microwave Anisotropy Probe (WMAP) satellite were released [1, 2, 3], several groups reported evidence for large-scale anomalies in the data that were suggestive of inconsistencies with the paradigm of a statistically isotropic Gaussian random field, or with the theoretical power spectrum. One widely discussed anomaly was an alignment of some of the low multipoles with a so-called Axis of Evil [4, 5, 6, 7]. Another was the CMB quadrupole  $C_2$ , which was unexpectedly low for the standard cosmological model [4]. A third anomaly was a difference in the observed CMB power spectrum between two hemispheres [8, 9, 10, 11, 12]. A variety of theoretical explanations for these anomalies have been proposed [13, 14, 15, 16, 17, 18, 19, 20].

The hemispherical power anomaly is the main subject of this paper. Its properties, as measured by Hoftuft *et al* [12], can be summarized as follows: (1) The direction of greatest power is  $\hat{\mathbf{p}} = (l, b) = (224^\circ, -22^\circ) \pm 24^\circ$ , near the South Ecliptic Pole. (2) The amplitude of large-scale fluctuations ( $\ell \leq 64$ ) is  $A = 0.072 \pm 0.022$  ( $3.3\sigma$ ), meaning that the amplitude of the CMB perturbations  $\Delta_T$  is 1.07 times the mean

value in the  $\hat{\mathbf{p}}$  direction, and 0.93 times the mean value in the  $-\hat{\mathbf{p}}$  direction.<sup>‡</sup> (3) The asymmetry is present in at least three of the WMAP bandpasses (41, 61, and 94 GHz), and in the foreground-suppressed internal linear combination (ILC) map [21]. (4) Only limited information is available about the  $\ell$ -dependence of the asymmetry. There are indications that it continues at least to  $\ell = 80$  [12] and perhaps beyond [11], but other analyses have found no evidence for an amplitude asymmetry at the first acoustic peak [28].

The large-scale anomalies are a difficult subject, in part because there is no clean way to assess their true significance. While the quoted significances sometimes exceed 99%, some of the analyses are *a posteriori*. In order to make progress it is necessary to consider the theoretical explanations for the large-scale anomalies, and ask what new predictions these models yield [22]. For some of the proposed ideas that only affect the low multipoles, the CMB temperature field has reached its cosmic variance limit and further tests must await future CMB polarization data with lower noise and foreground residuals than WMAP. On the other hand, one class of models intended to explain the hemispheric power asymmetry make a host of new predictions testable with near-future or even current data. Erickcek *et al* [20] recently proposed a variation on the curvaton model [23, 24, 25, 26]: an inflaton  $\phi$ , which dominates the energy density during inflation, and a curvaton  $\sigma$ , which is effectively massless during inflation but decays after inflation ends to produce an energy density  $\propto \sigma^2$ .<sup>§</sup> In the variation of Erickcek *et al* [20], there was initially a large-scale gradient in  $\sigma$ ; since the primordial density perturbations are proportional to  $\delta(\sigma^2) \approx 2\bar{\sigma}\delta\sigma$  (assuming they are sourced by the curvaton; inflaton-induced density perturbations may also be significant), the amplitude of the density perturbations is statistically inhomogeneous, being proportional to the local value of  $|\bar{\sigma}|$ . A uniform gradient in the background field  $\bar{\sigma}$  then results in a dipole modulation of the CMB power spectrum in the direction of some unit vector  $\hat{\mathbf{p}} \parallel \nabla|\bar{\sigma}|$ , with an amplitude given by the fractional variation of  $\bar{\sigma}$  across our horizon volume.

As noted by Erickcek *et al* [20], the proposed model could be tested with future CMB data, especially with the upcoming *Planck* satellite, which will observe many more modes in the CMB than WMAP and should definitively confirm or reject the power spectrum asymmetry possibly seen by WMAP. (The model could possibly even be constrained further by using the higher multipoles in WMAP.) However such a model also makes predictions for large-scale structure: since the dipole asymmetry in the power spectrum varies slowly with wavenumber  $k$ , it generates a large-scale gradient in the amplitude of fluctuations  $\sigma_8$ . Thus the abundance of rare massive haloes should contain a spatial gradient. Thus the angular distribution of these massive haloes (or any luminous tracer thereof) should contain a dipole in the same direction  $\hat{\mathbf{p}}$  singled out by the CMB. Moreover, using the measured dipole anisotropy of the CMB power spectrum and our knowledge of the halo mass function, one can specifically predict the amplitude of these features as a function of redshift and halo mass. Such features, if detected with the correct dependences and alignment of the CMB, would be a smoking gun for new physics, whereas nondetection of the large scale structure dipole would rule out the simplest version of the Erickcek *et al* [20]

<sup>‡</sup> The asymmetry in the  $C_\ell$ s is twice this,  $\pm 14$  percent, since the power spectrum is the square of the amplitude.

<sup>§</sup> We have assumed here that the curvaton never contributes more than a small fraction of the energy density of the Universe; this is true in the Erickcek *et al* model, but it is not true in all curvaton models.

model.

The purpose of this paper is two-fold: first, to develop the formalism to test for a spatial gradient in  $\sigma_8$ ; and second, to apply this formalism to the current large-scale structure data and measure  $\nabla\sigma_8$ . Our primary motivation was the Erickcek *et al* [20] model, but we note that any early-Universe explanation for the hemispherical asymmetry is likely to produce  $\nabla\sigma_8$  and hence our measurement should provide tight constraints. [We note that a statistically anisotropic power spectrum  $P(\mathbf{k})$  cannot produce the dipole signature seen by Refs. [8, 10, 11, 12] because it is invariant under  $\mathbf{k} \rightarrow -\mathbf{k}$ .] For our measurement of  $\nabla\sigma_8$ , we use the high-redshift spectroscopic quasars from the Sloan Digital Sky Survey (SDSS) as described below.

## 2. Theory

### 2.1. The model

The Erickcek *et al* [20] model produces a local primordial fluctuation amplitude  $\Delta_{\mathcal{R}}(k)$  that has a spatial gradient:

$$\Delta_{\mathcal{R}}(k, \mathbf{r}) = \Delta_{\mathcal{R}}(k, 0) \left( 1 + \mathbf{p} \cdot \frac{\mathbf{r}}{r_{\text{ls}}} \right), \quad (1)$$

where  $r_{\text{ls}}$  is the distance to the surface of last scattering. This normalization was chosen so that in observations of the CMB the amplitude of variations is modulated by a factor  $1 + A \cos \theta$ , where  $\theta$  is the angle between the gradient direction  $\hat{\mathbf{p}}$  and the line of sight  $\hat{\mathbf{n}}$  and  $A = |\mathbf{p}|$  (with minor modifications due to reionization; see below). Constraints in this paper will be reported on the amplitude  $A$  and direction  $\hat{\mathbf{p}}$  of this modulation.

### 2.2. Effect on CMB

A large-scale gradient in the primordial fluctuation amplitude will produce a corresponding dipole in the CMB anisotropies. On small scales, where the added power from the integrated Sachs-Wolfe (ISW) effect and reionization are negligible, the local CMB power spectrum in a particular region of the sky  $\hat{\mathbf{n}}$  is

$$C_l^{\text{loc}}(\hat{\mathbf{n}}) = C_l e^{-2\Delta\tau(\mathbf{n})} (1 + \mathbf{p} \cdot \hat{\mathbf{n}})^2, \quad (2)$$

where  $\Delta\tau(\mathbf{n})$  is the change in optical depth in direction  $\mathbf{n}$  relative to the reference model. There is motivation for such a term to exist: if  $\sigma_8$  is larger in some regions than others, the regions of higher  $\sigma_8$  will form more early galaxies and we expect that they will reionize first. In the case where  $A$  is small, we may write:

$$C_l^{\text{loc}}(\hat{\mathbf{n}}) \approx C_l (1 + 2X \mathbf{p} \cdot \hat{\mathbf{n}}), \quad (3)$$

where the factor

$$X = 1 - \frac{r_{\text{ri}}}{r_{\text{ls}}} \frac{d\tau}{d \ln \sigma_8} \quad (4)$$

accounts for the increase of  $\sigma_8$  and hence  $\tau$  in direction  $+\hat{\mathbf{p}}$  versus  $-\hat{\mathbf{p}}$ , and the factor  $r_{\text{ri}}/r_{\text{ls}}$  accounts for the fact that the reionization surface is closer to us than the last scattering surface. Reionization occurs during the matter-dominated era when the optical depth depends on the scale factor at reionization  $a_{\text{ri}}$  via  $\tau \propto a_{\text{ri}}^{-3/2}$ . Therefore

$$X = 1 + \frac{3\tau r_{\text{ri}}}{2r_{\text{ls}}} \frac{d \ln a_{\text{ri}}}{d \ln \sigma_8}. \quad (5)$$

From the WMAP 5-year parameter constraints we know that  $\tau = 0.087 \pm 0.017$  [27] and  $3\tau r_{\text{ri}}/(2r_{\text{iss}}) = 0.092 \pm 0.021$ . One does not know  $d \ln a_{\text{ri}}/d \ln \sigma_8$ , as this depends sensitively on astrophysics. If reionization is caused by massive haloes, so that the dependence of the reionization redshift on  $\sigma_8$  is dominated by changes in the halo mass function, then we have  $a_{\text{ri}} \propto \sigma_8^{-1}$  because the halo mass function during the matter-dominated era is invariant under rescalings of  $a$  and  $\sigma_8$  that hold  $\sigma_8 a$  constant. This case gives  $d \ln a_{\text{ri}}/d \ln \sigma_8 = -1$ , i.e. higher  $\sigma_8$  gives earlier reionization. This number will be increased somewhat (i.e. made closer to zero) if one takes into account that at earlier times more photons are required to reionize the intergalactic medium because of recombinations. It will also be increased if higher  $\sigma_8$  implies a larger clumping factor, or more minihaloes that act as a sink for ionizing photons. While the astrophysics is unclear at present, we are fortunate that the multiplying factor of 0.09 renders  $X$  relatively insensitive to the details: the  $d \ln a_{\text{ri}}/d \ln \sigma_8 = -1$  case would imply  $X \approx 0.91$ , whereas the extreme case of assuming reionization at an epoch independent of  $\sigma_8$  would give  $X = 1$ . For practical purposes  $X = 1$  with negligible correction (unless the anisotropy is measured at  $\geq 10\sigma$ ) and we will not consider it further.

The power asymmetry analysis of Ref. [10] using the 3-year WMAP data and a range of scales  $2 \leq \ell \leq 40$  found an asymmetry of  $A = 0.114$  with the direction  $\hat{\mathbf{p}}$  given by  $(l, b) = (225^\circ, -27^\circ)$  and 99.1% significance. A more recent analysis by Hansen *et al* [11] using the 5-year WMAP data and going out to  $\ell_{\text{max}} = 600$  finds preliminary evidence for a direction of  $(l, b) = (226^\circ \pm 10^\circ, -17^\circ \pm 10^\circ)$  at 99.6% significance. Very recently, Hoftuft *et al* [12] search the range of scales up to  $\ell_{\text{max}} = 64$  and find a  $3.3\sigma$  dipole,  $A = 0.072 \pm 0.022$  with  $\hat{\mathbf{p}} = (l, b) = (224^\circ, -22^\circ) \pm 24^\circ$  in the WMAP 5-year ILC map. In the 61 GHz band, they carried the analysis up to  $\ell_{\text{max}} = 80$  and found  $A = 0.070 \pm 0.019$  ( $3.7\sigma$ ) with  $\hat{\mathbf{p}} = (235^\circ, -17^\circ) \pm 22^\circ$ .

An analysis by Donoghue & Donoghue [28] of the first acoustic peak finds a power spectrum asymmetry of  $\eta = 2A = 0.02 \pm 0.02$  at  $\ell \sim 220$  (assuming the direction  $\hat{\mathbf{p}}$  given by Hansen *et al* [9]), implying  $A < 0.03$  at  $2\sigma$ . This is in tension with the low- $\ell$  results, and motivates further analysis to reduce the uncertainty on  $A$ .

One way to reduce the error bars on the CMB method would be to extend it to higher  $\ell$  where there are more modes available to constrain the hemispherical asymmetry. The number of CMB modes available is proportional to  $\ell_{\text{max}}^2$  and so we would expect the uncertainty in  $A$  to decrease as  $\propto \ell_{\text{max}}^{-1}$ ; the Hoftuft *et al* uncertainties are in rough agreement with this expectation. Extrapolating this scaling, an analysis making use of the full WMAP data out to  $\ell_{\text{max}} \sim 500$  should be able to reach  $\sigma(A) \sim 0.003$  if other sources of uncertainty (e.g. unmasked point sources) can be overcome. We believe such an analysis would be very helpful, especially to constrain the scale dependence of the asymmetry. However, in this paper we take an independent approach with similar sensitivity: we look for an asymmetry in large-scale structure.

### 2.3. Effect on large scale structure

The large-scale gradient in  $\sigma_8$  also has an effect on the abundance of massive haloes and any objects that occupy them. For objects at a distance  $r$ , the observed 2-dimensional number density varies across the sky as:

$$\frac{\delta N}{N}(\hat{\mathbf{n}}) = \frac{r}{r_{\text{iss}}} \frac{\partial \ln N}{\partial \ln \sigma_8} \mathbf{p} \cdot \hat{\mathbf{n}}. \quad (6)$$

Our major problem here is determining  $\partial \ln N / \partial \ln \sigma_8$ . This problem has been considered in the context of non-Gaussianity searches [29, 30, 31], which found

$$\frac{\partial \ln N}{\partial \ln \sigma_8} = \delta_c(b-1), \quad (7)$$

where  $\delta_c = 1.69$  and  $b$  is the bias. This formula assumed a universal mass function, as found in simulations, and assumed that the objects under study have a halo occupation distribution (HOD) that depends only on halo mass. The dependence on  $\sigma_8$  makes conceptual sense: very massive objects are highly biased  $b(M) \gg 1$ , and their abundance increases rapidly with  $\sigma_8$ ; low-mass objects are antibiased  $b(M) < 1$  and are less abundant at high  $\sigma_8$  because they will have merged; and the overall abundance of dark matter particles with  $b = 1$  does not depend on  $\sigma_8$ . If only recent major mergers are occupied, then there is an additional suppression in Eq. (7) because massive haloes formed earlier if  $\sigma_8$  is increased. In the extended Press-Schechter (ePS) formalism,  $b-1$  should then be replaced by  $b-1-\delta_c^{-1} = b-1.59$  [31]; this exponent has been confirmed by re-scaling of simulations [32, 33] which suggest that during the matter-dominated era the  $\sigma_8$  dependence for recent major mergers ranges between  $b-1.59$  and  $b-1.65$  depending on the remnant mass and progenitor mass ratio [31]. In the case of the SDSS quasars it is not clear what is the correct way to populate haloes and so both limiting cases ( $b-1$  and  $b-1.6$ ) must be considered. In practice the bias of the high-redshift quasars is so large ( $\sim 10$ ) that the difference between these cases is negligible.

In practice the quasars occupy a range of redshifts and the observed dipole is a superposition:  $\delta N/N = \mathbf{d} \cdot \hat{\mathbf{n}}$ , where

$$\mathbf{d} = \delta_c \mathbf{p} \int \frac{r(z)}{r_{\text{ls}}} [b(z)-1] P(z) dz, \quad (8)$$

with the replacement  $b-1 \rightarrow b-1.6$  in the recent major merger case. Here  $P(z)$  is the redshift distribution of the quasars, normalized to unity:  $\int P(z) dz = 1$ .

### 3. Application to SDSS quasars

We now turn to empirical determination of  $\nabla \sigma_8$  using the above formalism.

#### 3.1. Choice of dataset

The high-redshift spectroscopic quasars from SDSS were chosen for this study because:

- (i) They are very distant, which means that the same spatial gradient in  $\sigma_8$  translates into a large absolute difference  $\Delta \sigma_8$  across the sky.
- (ii) They have wide angular coverage, which minimizes large-scale structure noise and provides the leverage in all three Cartesian coordinates necessary to measure a dipole.
- (iii) They are highly biased, so that a small change in  $\sigma_8$  is amplified into a much larger change in the number density of quasars.
- (iv) They have very good rejection of Galactic objects (e.g. halo stars), which could otherwise produce spurious large angular scale modulation and potentially correlate with foreground residuals in the CMB maps.
- (v) The relative photometric calibration of SDSS is understood at the percent level (although this degrades to a few percent in the  $u$  band).

- (vi) The number density of  $\sim 1 \text{ deg}^{-2}$ , while not impressive, is acceptable given the above advantages. Given the choice between high number density of nearby objects with a weak dependence on  $\sigma_8$ , or a low number density of distant objects that depends strongly on  $\sigma_8$ , our preference is for the latter since Nature does much of the work to suppress systematics.

There are other data sets that could have been used instead to trace large-scale structure at moderate to high redshifts. Possibilities include:

- (i) *Luminous red galaxies*: These are abundant but current large-scale maps go out to only  $z < 0.7$  [34], or 18% of the distance to the last-scattering surface, and the typical bias is small,  $b \sim 1.8$ .
- (ii) *Photometrically selected quasars*: These have a higher number density ( $\sim 50 \text{ deg}^{-2}$ ), but at present the UV-based selection algorithms restrict this sample to  $z < 2.5$  [35]. The bias is also much lower at these redshifts,  $b \sim 2.3\text{--}2.8$  [36].
- (iii) *Hard X-ray background*: The 2–8 keV HEAO map has excellent sky coverage and perhaps the best-constrained dipole [37], but the redshift distribution is poorly known and the estimated bias is small and may be consistent with  $b = 1$  [38], so it is not known how sensitive this is to  $\nabla\sigma_8$ .
- (iv) *Radio sources*: The NRAO VLA Sky Survey [39] sample has been used to probe the moderate redshift range  $z \sim 1$ , but its redshift distribution is still under debate [40, 41, 36]. More importantly the maps exhibit declination-dependent striping which is almost certainly a systematic artifact and precludes determination of the dipole [42].
- (v) *Lyman- $\alpha$  forest*: A different approach would be to measure a fluctuation amplitude from the Lyman- $\alpha$  forest at the same redshift but with sightlines in different parts of the sky. The differential nature of the measurement would suppress many of the astrophysical systematics in e.g.  $P(k)$  measurement from the Lyman- $\alpha$  forest. This approach was not taken here because the analysis would be much more complicated, but given the large number of observed modes in the Lyman- $\alpha$  forest it should be considered in the future.

### 3.2. Data description

The high-redshift quasars used here are obtained from the SDSS. The SDSS drift-scans the sky in five bands (*ugriz*) [43] under photometric conditions [44, 45] using a 2.5-meter optical telescope [46] with 3 degree field of view camera [47] located in New Mexico, USA [44]. The photometric and astrometric calibration of the SDSS and the quality assessment pipeline are described by Refs. [48, 49, 50, 51, 52]. Bright galaxies [53], luminous red galaxies (LRGs) [54], and quasar candidates [55] are selected from the SDSS imaging data for spectroscopic follow-up [56].

The spectroscopic quasar sample used here was based on colour selection criteria described in Richards *et al* [55]. The algorithm selects quasar candidates down to a limiting magnitude of  $i = 19.1$  or  $20.2$  (for high- $z$  quasars); a discussion of its completeness and efficiency can be found in Richards *et al* [55]. A statistical sample of confirmed quasars was generated by Shen *et al* [57], who removed from the sample regions with early versions of the target selection algorithm. Shen *et al* imposed a further cut on the quality of the imaging data used for target selection, dividing it into “good” and “bad” fields. As a test for systematics, their quasar clustering analysis was repeated with both the “good” fields only, and with “all” fields.

The Shen *et al* [57] sample has a minimum redshift  $z \geq 2.9$ , due to the need in target selection to avoid Galactic late A/early F stars which have similar broadband colours to quasars at  $z \sim 2.7$ , and most objects are at  $z < 4.5$  due to the declining luminosity function and increasing luminosity distance at high redshift. There are 4426 quasars in the “all” sample (4041 deg<sup>2</sup>) and 3846 in “good” (3506 deg<sup>2</sup>). The quasars undersample the density field at all scales, so that their power spectrum is dominated by Poisson noise.

Shen *et al* specify a mask, which we have imported into MANGLE [58, 59, 60]. MANGLE is a suite of computer programmes widely used in the large-scale structure community to manipulate complex masks, which it typically represents as spherical polygons (i.e. regions bounded by circular arcs). MANGLE can carry out simple tasks on these masks such as unions and spherical harmonic transform computation; most importantly for us, it can generate catalogues of random points from within a given mask.

For this analysis, we have further split the quasars into a  $2.9 \leq z < 3.5$  sample, and a  $3.5 \leq z < 4.5$  sample.

### 3.3. Dipole anisotropy estimator

In general the quasar density in a given direction can be written as:

$$\frac{\delta N}{\bar{N}}(\hat{\mathbf{n}}) = \mathbf{d} \cdot \hat{\mathbf{n}} + \sum_i k_i t_i(\hat{\mathbf{n}}) + C. \quad (9)$$

Here  $\bar{N}$  is the mean number of quasars per steradian,  $\delta N$  is its fluctuation,  $\mathbf{d}$  is the dipole (which we wish to measure),  $t_i(\hat{\mathbf{n}})$  are possible systematics templates in the quasar maps (e.g. the extinction map),  $k_i$  are the sensitivities of the quasar densities to these systematics, and  $C$  is a mean offset over which we must marginalize since the true mean number density of quasars is not known. This equation can be simply written as:

$$\frac{\delta N}{\bar{N}}(\hat{\mathbf{n}}) = \mathbf{x} \cdot \mathbf{T}(\hat{\mathbf{n}}), \quad (10)$$

where we have defined the vectors  $\mathbf{x} = (\mathbf{d}, k_i, C)$  and  $\mathbf{T}(\hat{\mathbf{n}}) = (\hat{\mathbf{n}}, t_i(\hat{\mathbf{n}}), 1)$ . If Poisson noise dominates the uncertainty in  $\delta N/\bar{N}$ , so that the inverse variance per steradian is  $\bar{N}$ , then the best linear unbiased estimator for  $\mathbf{x}$  is obtained via:

$$\hat{\mathbf{x}} = \mathbf{F}^{-1} \mathbf{g}, \quad (11)$$

where

$$g_i = \int T_i(\hat{\mathbf{n}}) \delta N(\hat{\mathbf{n}}) d^2 \hat{\mathbf{n}} \quad (12)$$

and the Fisher matrix is

$$F_{ij} = \bar{N} \int T_i(\hat{\mathbf{n}}) T_j(\hat{\mathbf{n}}) d^2 \hat{\mathbf{n}}. \quad (13)$$

The covariance matrix of this estimator is  $\text{Cov}(\hat{\mathbf{x}}) = \mathbf{F}^{-1}$ .

The above equations are integrals over the survey area, but quasars are discrete objects and so it is easiest to replace the above results with summations over the quasars and random catalogues. We can do this by writing:

$$g_i = \sum_{\text{D}} T_i(\hat{\mathbf{n}}_{\text{D}}) - \frac{N_{\text{D}}}{N_{\text{R}}} \sum_{\text{R}} T_i(\hat{\mathbf{n}}_{\text{R}}), \quad (14)$$

where  $\hat{\mathbf{n}}_D$  are the positions of the quasars in the real data,  $\hat{\mathbf{n}}_R$  are the positions of randomly generated points in the survey mask, and  $N_D$  and  $N_R$  are the number of quasars (data) and random points respectively. The Fisher matrix is:

$$F_{ij} = \frac{N_D}{N_R} \sum_R T_i(\hat{\mathbf{n}}_R) T_j(\hat{\mathbf{n}}_R). \quad (15)$$

We have generated random catalogues containing  $N_R = 10^6$  points using MANGLE’s RANSACK task [60], which is more than sufficient as their residual noise is negligible ( $< 1\%$ ) compared to the actual data.

### 3.4. Measured dipole anisotropy

We have applied the estimator Eq. (11) to the high- $z$  SDSS quasar data. Four versions of the fit were done, which differed by using the ‘all’ or ‘good’ field mask, and by either including or not including the dust template. The dust template used is the predicted reddening map of  $E(B-V)$  by Schlegel *et al* [61] as a systematics template, which is appropriate if the extinction correction is not perfect. Errors correlated with the map could arise due to errors in the assumed extinction law  $A_\lambda/E(B-V)$ , miscalibration of the  $E(B-V)$  scale, or unusual quasar colours affecting the extinction correction (e.g. if a bright emission line lies at the red end of the bandpass then the band-averaged correction may overcorrect for the extinction). The template amplitude  $k_{E(B-V)}$  could plausibly be positive or negative depending on the amount of over- or undercorrection in different bands. Because most of the data lie in the Northern Galactic cap, there is a strong degeneracy between the component of the dipole perpendicular to the Galactic plane and the extinction correction,  $\text{Corr}(k_{E(B-V)}, d_z) = 0.65$ : either increasing  $d_z$  or decreasing  $k_{E(B-V)}$  would have the effect of enhancing the number density of objects near the NGP. This degeneracy could be broken in the future with more data in the southern patch. For the ‘all’ mask, we find template amplitudes of  $k_{E(B-V)} = -1.6 \pm 1.2$  (low- $z$ ) and  $k_{E(B-V)} = -3.5 \pm 1.6$  (high- $z$ ); these become  $-1.0 \pm 1.3$  and  $-3.6 \pm 1.7$  for the ‘good’ mask. We regard the fits with the dust template as our primary result because correlation with dust is marginally detected at  $> 2\sigma$  in one of the redshift slices.

The resulting dipole moments in the Galactic coordinate system are shown in Table 1. To be explicit, we have placed the  $x$  direction toward the Galactic centre, the  $y$  direction in the plane at  $l = 90^\circ$ , and the  $z$  direction toward the North Galactic Pole (NGP). Using the ‘all’ mask and no dust template, one finds an intriguing  $2.9\sigma$  hint of  $d_z > 0$  in the low- $z$  quasars. However the fact that the preferred direction is very near the NGP ( $b = 80^\circ$ ) is suspicious, and indeed this signal goes away if one restricts to ‘good’ fields (significance drops to  $1.8\sigma$ ), uses the dust template ( $1.4\sigma$ ), or both ( $0.8\sigma$ ).

In all cases the uncertainty in the dipole determination is at least several percent, so no correction for our own peculiar velocity  $v/c = 0.0012$  [2] is necessary.

In addition to the Galactic dust, a second concern would be large-angle calibration errors, which can involve either ‘grey’ errors (in which all five bands move together) or colour calibration errors. We may do a simple assessment of the grey calibration error by comparing the target selection photometry to the ubercalibrated photometry [50] obtained from the Shen *et al* [57] catalogue, in  $i$  band as used for the flux cut. The ubercalibrated photometry is estimated to be accurate to 1%. We compare the



**Table 1.** The observed dipole anisotropies, in Galactic Cartesian coordinates, for the various quasar samples and for the dust template either included (Yes) or not (No).

Sample	Dust?	$10^2 d_x$	$10^2 d_y$	$10^2 d_z$
2.9–3.5, all	No	$-1.3 \pm 5.1$	$2.6 \pm 6.5$	$16.3 \pm 5.6$
3.5–4.5, all	No	$-0.5 \pm 6.7$	$-4.2 \pm 8.4$	$-1.9 \pm 7.3$
2.9–3.5, all	Yes	$1.1 \pm 5.4$	$-0.7 \pm 6.9$	$10.0 \pm 7.4$
3.5–4.5, all	Yes	$4.8 \pm 7.1$	$-11.7 \pm 9.0$	$-15.9 \pm 9.6$
2.9–3.5, good	No	$-2.3 \pm 5.5$	$2.8 \pm 6.9$	$10.8 \pm 6.0$
3.5–4.5, good	No	$3.2 \pm 7.1$	$-2.7 \pm 9.0$	$-7.4 \pm 6.8$
2.9–3.5, good	Yes	$3.9 \pm 5.8$	$0.6 \pm 7.5$	$6.6 \pm 7.9$
3.5–4.5, good	Yes	$8.7 \pm 7.6$	$-10.5 \pm 9.6$	$-21.9 \pm 10.2$

magnitude difference  $\Delta i = i_{\text{target}} - i_{\text{ubercal}}$  for each quasar, and do an unweighted least-squares fit

$$\Delta i = \Delta i_0 + \mathbf{m} \cdot \hat{\mathbf{n}}, \quad (16)$$

where  $\mathbf{m}$  is the calibration dipole and  $\mathbf{n}$  is the position of that quasar. We remove two (possibly variable) objects from the fit that were undetected in the ubercalibrated photometry. The resulting dipole is  $10^3 \mathbf{m} = (0 \pm 3, -10 \pm 3, -3 \pm 3)$ . We have also tried clipping the distribution of  $\Delta i$  at the 1st and 99th percentiles (i.e. quasars below the 1st percentile were moved to the 1st percentile value before fitting). The resulting dipole is  $10^3 \mathbf{m} = (0.6 \pm 0.8, -3.5 \pm 0.9, 0.5 \pm 0.8)$ . The implied dipole due to changes in the  $i$  band calibration would be 1.8 times this, since the slope of the cumulative quasar luminosity function for this redshift range and the  $i < 20.2$  flux cut is  $\sim 1.8$  (see e.g. Fig. 13 of Ref. [62]). Even the 1% dipole obtained from the first fit would lead to a calibration-induced dipole of 1.8% in the density of quasars, which is much less than our 5–10% statistical errors. (But do note that both calibration dipoles are inconsistent with zero, giving  $\chi^2$  for 3 degrees of freedom of 12 and 19, respectively.)

Colour calibration is potentially nastier since quasars are selected by cuts that run through the quasar locus, and hence could result in completeness variations if there is a relative colour calibration error. Our best test of the colour-dependent calibration will be the agreement (or lack thereof) of the dipoles inferred from different redshift ranges where the colour selection is very different. However, we can also test for this by computing the dipole of the reddening-corrected colours, i.e. doing an unweighted least-squares fit

$$u - g = (u - g)_0 + \mathbf{m} \cdot \hat{\mathbf{n}}, \quad (17)$$

where  $(u - g)_0$  and the vector  $\mathbf{m}$  represent 4 free parameters. We use all of the quasars in the  $2.9 \leq z < 4.5$  range for this test. Since some quasars are undetected in  $u$ , we clip the distribution at the 5th and 95th percentiles in  $u - g$  before performing the fit (i.e. quasars below the 5th percentile were moved to the 5th percentile value before fitting). One can then test for whether  $\mathbf{m} \neq 0$ , which would possibly indicate a dipole variation in colour calibration across the sky. We report the  $\chi^2$  value,  $\mathbf{m} \cdot \text{Cov}(\mathbf{m}) \mathbf{m}$ , and the probability to exceed  $P(> \chi^2)$ . Similar tests can be done for the other colours, and for the extinction-corrected apparent magnitudes of the quasars. The magnitudes and colours used are those used for the target selection (TARGET PSF photometry), which were obtained via matching to the SDSS DR5 quasar catalogue [63]. The results are shown in Table 2. As one can see, the  $\chi^2$  values for these fits are all acceptable,

**Table 2.** The  $\chi^2$  values and probabilities to exceed for searches for dipole gradients in the quasar colour.

Quantity	$\chi^2(\mathbf{m})$	$P(> \chi^2)$
$u - g$	3.05	0.38
$g - r$	4.61	0.20
$r - i$	9.10	0.028
$i - z$	2.84	0.42
$u - r$	2.22	0.53
$g - i$	4.53	0.21
$r - z$	6.29	0.10
$u - i$	1.58	0.66
$g - z$	4.92	0.18
$u - z$	1.36	0.72

with one possible exception ( $r - i$ ,  $p = 0.028$ ). Given that we searched 10 directions in colour space, the presence of this one anomalously high  $\chi^2$  value is not a concern.

### 3.5. Quasar bias and final results

In order to convert the measured dipole anisotropy into a constraint on the perturbation amplitude asymmetry  $A$  using Eq. (8), we need to know the bias of the quasars. If one assumes a cosmological model, one may obtain this by fitting to the correlation function. Here we use the WMAP5+BAO cosmology, based on combining the WMAP 5-year power spectrum [64] with the SDSS+2dF baryon acoustic oscillation (BAO) constraint [65]; this combination yields  $\sigma_8 = 0.807 \pm 0.027$  [27].

If one has a measured correlation length  $r_0$  and an assumed cosmology, the bias can be obtained by the formula  $b = [\xi_m(r_0)]^{-1/2}$ , where  $\xi_m(r)$  is the matter correlation function. We have used the linear correlation function using the Eisenstein & Hu transfer function [66] since in the regime of interest the nonlinear corrections as estimated by the Smith *et al* [67] prescription are  $< 1\%$ , i.e. negligible. The correlation lengths that Shen *et al* [57] measured from the “good” fields are  $17.91 \pm 1.50 h^{-1} \text{ Mpc}$  ( $2.9 \leq z < 3.5$ ) and  $25.22 \pm 2.50 h^{-1} \text{ Mpc}$  ( $z \geq 3.5$ ). These correlation lengths imply  $b = 8.9 \pm 0.7$  ( $z = 3.2$ ) and  $b = 15.3 \pm 1.7$  ( $z = 4.0$ ), where the uncertainty is simply propagated from  $r_0$ . There is some additional uncertainty in these numbers from nonlinear biasing, from the cosmological parameters, and because the correlation length does not appear to be constant with redshift. We note that the one-halo term in the correlation function cannot contaminate these measurements because Shen *et al* only fit projected separations  $r_p > 4h^{-1} \text{ Mpc}$ . Integrating over the redshift bin assuming a constant correlation length  $r_0$ , we turn Eq. (8) into:

$$\mathbf{d} = \begin{cases} (5.8 \pm 0.6)\mathbf{p} & \text{low-}z \text{ slice} \\ (11.5 \pm 1.4)\mathbf{p} & \text{high-}z \text{ slice} \end{cases} \quad (18)$$

Using the central value of the conversion from the observed quasar dipole into  $\mathbf{p}$ , and the conservative (good + dust marginalization) dipole estimates, we obtain  $10^2 \mathbf{p} = (0.7 \pm 1.0, 0.1 \pm 1.3, 1.1 \pm 1.4)$  for the low- $z$  slice and  $(0.8 \pm 0.7, -0.9 \pm 0.8, -1.9 \pm 0.9)$  for the high- $z$  slice. Combining the two results with the usual weighting by their inverse-covariance matrix gives

$$10^2 \mathbf{p} = (0.73, -0.61, -1.01), \quad (19)$$

with covariance:

$$10^4 \text{Cov}_{(\mathbf{p})} = \begin{pmatrix} 0.306 & -0.225 & -0.122 \\ -0.225 & 0.497 & 0.227 \\ -0.122 & 0.227 & 0.560 \end{pmatrix}. \quad (20)$$

In the optimal determination of the central value (Eq. 19) the low-redshift slice receives 30% of the weight and the high-redshift slice receives 70%, i.e.

$$\mathbf{p} = w_{\text{lo-z}} \mathbf{p}_{\text{lo-z}} + w_{\text{hi-z}} \mathbf{p}_{\text{hi-z}} \quad (21)$$

with  $w_{\text{lo-z}} = 0.30$  and  $w_{\text{hi-z}} = 0.70$ .

The difference between the inferred dipoles from the two samples is  $\mathbf{p}_{\text{lo-z}} - \mathbf{p}_{\text{hi-z}} = (-0.1, 1.0, 3.1) \times 10^{-2}$ ; based on the covariance matrix for the difference, the  $\chi^2$  is 3.03 (3 degrees of freedom;  $P = 0.39$ ) so we conclude the dipoles measured from the two redshift slices are consistent. There is no detection of any global dipole ( $\chi^2 = 2.80$ ;  $P = 0.42$ ).

If one fixes  $\hat{\mathbf{p}}$  to be in the specific direction  $(l, b) = (225^\circ, -27^\circ)$  identified by Eriksen *et al* [10], then the magnitude  $A$  of  $\mathbf{p}$  is found to be  $10^2 A = -0.18 \pm 0.44$ , i.e.  $-0.0105 < A < 0.0070$  at 95% confidence ( $-0.0132 < A < 0.0097$  at 99%). A direction-independent upper limit can be set by a Bayesian analysis analogous to Eriksen *et al* : by placing a uniform prior on the magnitude  $A$  and direction  $\hat{\mathbf{p}}$ , we can construct a marginalized likelihood function for  $A$ :

$$\mathcal{L}(A) \propto \int \exp \left[ -\frac{1}{2} (\mathbf{p} - \mathbf{p}_{\text{best}}) \cdot \text{Cov}^{-1} (\mathbf{p} - \mathbf{p}_{\text{best}}) \right] d^2 \hat{\mathbf{p}}, \quad (22)$$

where  $\mathbf{p} = A \hat{\mathbf{p}}$ . We find that 95% of the Bayesian posterior distribution is at  $A < 0.019$  and 99% at  $A < 0.026$ .<sup>||</sup>

In reality the above computations should take into account the uncertainty in the estimator response, Eq. (18), which derives from the uncertainty in the quasar bias. In particular it is possible to circumvent the above limits on  $\mathbf{p}$  if the quasar bias is smaller than central values. For our chosen weights, Eq. (21), the response of our estimator is:

$$\mathbf{p}_{\text{estimated}} = (1.00 \pm 0.09) \mathbf{p}_{\text{actual}}, \quad (23)$$

i.e. there is a 9% ( $1\sigma$ ) calibration uncertainty. We can include this uncertainty in a Bayesian analysis by incorporating a calibration factor  $\chi$  with a prior at  $1.00 \pm 0.09$ . For the fixed-direction case (i.e. where  $\hat{\mathbf{p}}$  is required to point in the Eriksen *et al* direction), one may define a marginalized likelihood function for  $A$ :

$$\mathcal{L}_{\text{marg}}(A) \propto \int e^{-(\chi A + 0.0018)^2 / 2(0.0044)^2} e^{-(\chi - 1)^2 / 2(0.09)^2} d\chi. \quad (24)$$

This gives  $-0.0108 < A < 0.0071$  (95%) or  $-0.0138 < A < 0.0100$  (99%). For the direction-independent case,

$$\mathcal{L}_{\text{marg}}(A) = \int \mathcal{L}_{\text{un-marg}}(\chi A) e^{-(\chi - 1)^2 / 2(0.09)^2} d\chi, \quad (25)$$

where the un-marginalized function  $\mathcal{L}_{\text{un-marg}}(A)$  is given by Eq. (22). Here we find that 95% of the Bayesian posterior distribution is at  $A < 0.020$  and 99% is at  $A < 0.027$ . These are negligible changes from the un-marginalized case.

<sup>||</sup> Since  $A$  is the magnitude of  $\mathbf{p}$ , in our direction-independent constraint it is required to be non-negative. In the fixed-direction constraint where  $\hat{\mathbf{p}}$  is fixed to the Eriksen *et al* direction, we allow for the possibility of negative  $A$ , i.e. a dipole opposite to that seen in the CMB.

We have also repeated the above analysis for the “all” fields. In this case, the correlation length measured by Shen *et al* [57] is shorter, so the biases are less:  $b = 8.0 \pm 0.9$  at  $z = 3.2$  and  $b = 13.5 \pm 1.6$  at  $z = 4.0$ . The dipole response is now

$$\mathbf{d} = \begin{cases} (5.1 \pm 0.7)\mathbf{p} & \text{low-}z \text{ slice} \\ (10.0 \pm 1.3)\mathbf{p} & \text{high-}z \text{ slice} \end{cases} . \quad (26)$$

The best-fit dipole and its covariance are:

$$10^2 \mathbf{p} = (0.40, -0.87, -0.52) \quad (27)$$

and

$$10^4 \text{Cov}_{(\mathbf{p})} = \begin{pmatrix} 0.351 & -0.259 & -0.141 \\ -0.259 & 0.571 & 0.261 \\ -0.141 & 0.261 & 0.643 \end{pmatrix} . \quad (28)$$

The weights in this case are  $w_{\text{lo-}z} = 0.30$  and  $w_{\text{hi-}z} = 0.70$ . Again the low- $z$  and high- $z$  slices are consistent ( $\chi^2 = 3.11$ ,  $P = 0.37$ ). There is also no detection of  $\mathbf{p}$ : the  $\chi^2$  for zero dipole is 1.34 ( $P = 0.72$ ). The calibration uncertainty is now 10%. After marginalizing over this, the fixed-direction constraint on  $A$  is  $-0.0073 < A < 0.0120$  (95%) or  $-0.0105 < A < 0.0153$  (99%), and the direction-independent constraint is  $A < 0.017$  (95%) or  $A < 0.025$  (99%).

### 3.6. Scale of constraint

Up until now we have assumed that the dipole anisotropy  $\hat{\mathbf{p}}$  is scale-invariant. However inflationary models usually predict slight deviations from scale invariance because as one approaches the end of inflation each e-fold is slightly different from the previous one. In order to constrain such models, we need to know the effective scale at which the quasars constrain  $\hat{\mathbf{p}}$ . Since the fluctuation amplitude is typically a smoothly varying function of the number of e-folds or equivalently of  $\ln k$ , only a rough estimate of  $k_{\text{eff}}$  is required.

We now obtain an estimate of the scale probed by the quasars, i.e. at what scale  $k_{\text{eff}}$  we have actually constrained the large-scale gradient of the fluctuation amplitude. The abundance of very massive haloes (i.e. well above the nonlinear mass) is roughly determined by the top-hat variance  $\sigma(R)$ , where  $M = 4\pi\bar{\rho}_{m0}R^3/3$  is the halo mass,  $\bar{\rho}_{m0}$  is the present-day matter density, and  $R$  is the comoving radius of the top-hat filter. This is:

$$\sigma^2(R) = \int \Delta^2(k) W^2(kR) d \ln k, \quad (29)$$

where the window function is  $W(x) = 3j_1(x)/x$ . Differentiating gives

$$\delta \ln \sigma(R) = \frac{\int \Delta^2(k) W^2(kR) \delta \ln \Delta(k) d \ln k}{\int \Delta^2(k) W^2(kR) d \ln k}. \quad (30)$$

This can be approximated as  $\delta \ln \sigma(R) = \delta \ln \Delta(k_{\text{eff}})$ , where:

$$\ln k_{\text{eff}} = \frac{\int \Delta^2(k) W^2(kR) \ln k d \ln k}{\int \Delta^2(k) W^2(kR) d \ln k}. \quad (31)$$

For the quasars, the minimum mass (which for a steep mass function can be taken as a typical mass) is  $M \sim (2-6) \times 10^{12} h^{-1} M_{\odot}$  [57], which gives  $k_{\text{eff}} = (1.3-1.8)h \text{ Mpc}^{-1}$ .

This range spans only 0.3 e-folds and in what follows we will thus take the central value,  $k_{\text{eff}} = 1.5h \text{ Mpc}^{-1}$ .

One can make a similar estimate for the Eriksen *et al* [10]; we find in Appendix A that  $k_{\text{eff}} = 0.0033h \text{ Mpc}^{-1}$ . The Eriksen *et al* constraint and our constraint are thus separated by 6.1 e-folds. This strongly constrains inflationary explanations for the power asymmetry: the asymmetry amplitude  $A$  must decrease by a factor of at least a few ( $\sim 7\%$  to  $< \sim 1\%$ ) during only 6.1 e-folds of expansion. This contrasts with the usual expectation from inflationary models of nearly scale-invariant spectra. One could imagine features in the inflationary potential that strongly break scale invariance; however there is no motivation to place such features in precisely this range of  $\ln k$ , and moreover such a model would have to avoid “breaking” the successes of standard inflation in predicting the CMB acoustic peak regime and large-scale galaxy clustering. A detailed study of such constraints is presented in a companion paper [69].

#### 4. Discussion

One of the intriguing products of the WMAP mission was the detection of large-scale anomalies in the CMB. If these anomalies are “real” (as opposed to statistical fluctuations or systematics) then their implications are revolutionary. The anomalies are thus in need of confirmation, both by future CMB data that could detect them at higher signal-to-noise ratio (e.g. *Planck*) and just as importantly by independent probes of the same underlying physics. In particular, the dipolar asymmetry in the CMB power spectrum reported by Eriksen *et al* [10], if interpreted as a spatial variation of the amplitude of primordial fluctuations, makes predictions not just for the CMB but also for large-scale structure: if there is truly a large-scale gradient  $\nabla\sigma_8$  across the Universe, then there should be a corresponding gradient in the number density of highly biased objects. We have searched for such a gradient using the SDSS  $z \geq 2.9$  quasars and found a null result. We find that the dipole  $\mathbf{p} = r_{\text{ISS}} \nabla \ln \sigma_8$  is no more than 0.027 (99% posterior probability), i.e. any smooth gradient in the amplitude of fluctuations is no more than 2.7 percent per present-day horizon radius. Tighter results are obtained if one forces the gradient to be in the same direction as that reported by Eriksen *et al* [10].

This result rules out the simple curvaton-gradient model [20] in which the power asymmetry in the CMB comes from a spatial gradient in the curvaton field of a two-field inflation model. Such a model requires  $p \approx 0.11$  to reproduce the reported  $\pm 11\%$  variation in the CMB amplitude across the sky [10]. This does not necessarily contradict the Eriksen *et al* [10] result: there may be more complicated models with a scale-dependent gradient in small-scale power, i.e. where the spatial gradient of the local  $\ln P(k)$  depends on  $k$ . Such models, including some that predict much smaller dipoles in the quasar distribution and the degree-scale CMB fluctuation amplitudes, are explored in the companion paper by Erickcek *et al* [69].

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## Appendix A. Effective scale of CMB power asymmetry

Eriksen *et al* [10] measured a dipole asymmetry in the CMB power spectrum in the range of multipoles  $2 \leq \ell \leq \ell_{\max} \sim 40$ . The upper limit  $\ell_{\max}$  was set by the requirement to be oversampled on the HEALPIX resolution 4 ( $N_{\text{side}} = 16$ ) pixelization scheme [68]. Our goal here is to estimate the effective wavenumber of the Eriksen *et al* measurement.

In the Sachs-Wolfe regime, and neglecting the small contribution from the integrated Sachs-Wolfe effect, the temperature perturbation is given by  $\Theta(\hat{n}) = -\zeta(r_{\text{ls}}\hat{n})/5$  where  $\zeta$  is the primordial curvature perturbation. Therefore the CMB power spectrum at multipole  $\ell$  is obtained by

$$C_\ell = \frac{4\pi}{25} \int j_\ell^2(kr_{\text{ls}}) \Delta_\zeta^2(k) d \ln k, \quad (\text{A.1})$$

where  $\Delta_\zeta^2(k)$  is the primordial curvature power spectrum. [See, e.g. Eqs. (5.27) and (5.39) of Ref. [70].] For smooth power spectra and  $\ell \gg 1$ , we can use the Wentzel-Kramers-Brillouin approximation for the spherical Bessel function,

$$j_\ell(x) \rightarrow x^{-1/2}(x^2 - \ell^2)^{-1/4} \cos \varphi(x) \quad (\text{A.2})$$

for  $x > \ell$ , where  $\varphi(x)$  is a rapidly oscillating phase. (For  $x < \ell$  the spherical Bessel function goes rapidly to zero.) With this approximation, and taking  $\langle \cos^2 \varphi(x) \rangle \rightarrow \frac{1}{2}$ ,  $C_\ell$  simplifies to

$$C_\ell \approx \frac{2\pi}{25\ell^2} \int y^{-1}(y^2 - 1)^{-1/2} \Delta_\zeta^2(k) d \ln k, \quad (\text{A.3})$$

where  $y = x/\ell = kr_{\text{ls}}/\ell$ . The fractional dipole asymmetry  $\delta \ln C_\ell$  of  $C_\ell$  is given by:

$$\delta \ln C_\ell \approx \frac{\int y^{-1}(y^2 - 1)^{-1/2} \Delta_\zeta^2(k) \delta \ln \Delta_\zeta^2(k) d \ln k}{\int y^{-1}(y^2 - 1)^{-1/2} \Delta_\zeta^2(k) d \ln k}. \quad (\text{A.4})$$

This equation can be simplified for the case of a near scale-invariant reference spectrum,  $\Delta_\zeta^2(k) = \text{constant}$ :

$$\delta \ln C_\ell \approx \frac{\int_{y=1}^{\infty} y^{-1} (y^2 - 1)^{-1/2} \delta \ln \Delta_\zeta^2(k) d \ln y}{\int_{y=1}^{\infty} y^{-1} (y^2 - 1)^{-1/2} d \ln y}. \quad (\text{A.5})$$

The integral in the denominator evaluates to 1 (with the substitution  $y = \sec \beta$ ).

In order to make further progress, we assume that  $\delta \ln \Delta_\zeta^2(k)$  varies smoothly with  $\ln k$ , as typical during inflation models, i.e. we assume:

$$\delta \ln \Delta_\zeta^2(k) = B_1 + B_2 \ln(k r_{\text{ISS}}) = B_1 + B_2 \ln \ell + B_2 \ln y. \quad (\text{A.6})$$

Only the  $\ln y$  part of the  $C_\ell$  integral is nontrivial,

$$\begin{aligned} \delta \ln C_\ell &\approx B_1 + B_2 \ln \ell + B_2 \int_{y=1}^{\infty} y^{-1} (y^2 - 1)^{-1/2} \ln y d \ln y \\ &= B_1 + B_2 \ln \ell + B_2 (1 - \ln 2). \end{aligned} \quad (\text{A.7})$$

{The integral over  $y$  can be solved by the substitution  $y = (1 - \rho^2)^{-1/2}$ , which turns it into  $-\frac{1}{2} \int_0^1 [\ln(1 - \rho) + \ln(1 + \rho)] d\rho$ . The  $\ln$  functions are then integrated by parts.}

The Eriksen *et al* [10] dipole asymmetry measurement fits a single amplitude for the power asymmetry across their entire range of  $\ell$ . In the limit where all multipoles up to  $\ell_{\text{max}}$  are signal-dominated and those above  $\ell_{\text{max}}$  are noise-dominated, the Eriksen *et al* asymmetry parameter  $A$  will receive equal weight from all modes, i.e.

$$A = \frac{1}{2} \langle \delta \ln C_\ell \rangle_{\text{modes}} \approx \frac{1}{2} [B_1 + B_2 \langle \ln \ell \rangle_{\text{modes}} + B_2 (1 - \ln 2)]. \quad (\text{A.8})$$

(The factor of 1/2 comes from the fact that Eriksen *et al* measured an amplitude asymmetry rather than a power asymmetry.) Since the number of modes in any region of the sky and any  $\ell$  range is proportional to  $\ell d\ell$ , we have, for  $\ell_{\text{max}} \gg 1$ ,

$$\langle \ln \ell \rangle_{\text{modes}} = \frac{\int_0^{\ell_{\text{max}}} \ell \ln \ell d\ell}{\int_0^{\ell_{\text{max}}} \ell d\ell} = \ln \ell_{\text{max}} - \frac{1}{2}. \quad (\text{A.9})$$

(The integral in the numerator is evaluated using repeated integration by parts.) Thus:

$$A = \frac{1}{2} \langle \delta \ln C_\ell \rangle_{\text{modes}} \approx \frac{1}{2} \left[ B_1 + B_2 \left( \ln \ell_{\text{max}} + \frac{1}{2} - \ln 2 \right) \right]. \quad (\text{A.10})$$

This is the same as the amplitude asymmetry  $\frac{1}{2} \delta \ln \Delta_\zeta^2(k_{\text{eff}})$  at the effective scale:

$$k_{\text{eff}} = \frac{e^{1/2} \ell_{\text{max}}}{2 r_{\text{ISS}}}, \quad (\text{A.11})$$

where  $e = 2.718...$  is the base of the natural logarithm.

For the specific case of Eriksen *et al*,  $r_{\text{ISS}} = 1.0 \times 10^4 h^{-1} \text{ Mpc}$  and  $\ell_{\text{max}} = 40$ , so  $k_{\text{eff}} = 0.0033 h \text{ Mpc}^{-1}$ . Similarly, for Hoftuft *et al* [12],  $k_{\text{eff}} = 0.0053 h \text{ Mpc}^{-1}$  ( $\ell_{\text{max}} = 64$ ) or  $k_{\text{eff}} = 0.0066 h \text{ Mpc}^{-1}$  ( $\ell_{\text{max}} = 80$ ).

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